

Cooperative Spectrum Sharing Relaying Protocols With Energy Harvesting Cognitive User

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Abstract—The theory of wireless information and power transfer in energy constrained wireless networks has caught the interest of researchers due to its potential in increasing the lifetime of sensor nodes and mitigate the environment hazards caused by conventional cell batteries. Similarly, the advancements in areas of cooperative spectrum sharing protocols has enabled efficient use of frequency spectrum between a licensed primary user and a secondary user. In this paper, we consider an energy constrained secondary user which harvests energy from the primary signal and relays the primary signal in exchange for the spectrum access. We consider Nakagami-m fading model and propose two key protocols, namely time-splitting cooperative spectrum sharing (TS-CSS) and power-sharing cooperative spectrum sharing (PS-CSS), and derive expressions for the outage probabilities of the primary and secondary user in decode-forward and amplify-forward relaying modes. From the obtained results, it has been shown that the secondary user can carry its own transmission without adversely affecting the performance of the primary user and that PS-CSS protocol outperforms the TS-CSS protocol in terms of outage probability over a wide range of Signal to noise ratio(SNRs). The effect of various system parameters on the outage performance of these protocols have also been studied.

Index Terms—Wireless Sensor Networks, Wireless Energy Harvesting, Cooperative Spectrum Sharing, Nakagami fading.

I. INTRODUCTION

WITH the proliferation in the deployment of Wireless Sensor Networks in fields like automated location monitoring, indoor localization and smart building initiatives, the demand for self-sustaining and long-running sensor nodes has increased [1]. Surveillance and tracking using sensor networks has become ubiquitous, where most of the sensor nodes run on coin cell batteries [2]. The need to eliminate the human intervention in replacement of the batteries and to make the sensor nodes more environment friendly has boosted the research in the field of wireless energy harvesting in sensor networks. Among the possible sources for energy, RF energy harvesting is of particular interest as RF signals are abundant in nature through various wireless technologies and have the capacity to carry information and energy simultaneously [3]. A capacity-energy function to evaluate the fundamental performance limits between the transport of power and information simultaneously through a single noisy line has been studied in [4], while the authors in [5] have

given the performance limits of MIMO broadcasting systems for simultaneous information and power transfer. In [4] and [5], it is assumed that the receiver is able to extract information and power from the same received signal. The authors in [6] have, however, addressed practical circuit limitations for this assumption and studied receiver architectures which employ dynamic power sharing (DPS) mechanisms to carry the information decoding and power transfer. Cooperative relaying schemes have been widely adopted in literature to enhance the performance of a communication system in terms of diversity, coverage extension etc. [7]–[11]. The relay might first decode the received information and then transmit, which is termed as decode and forward (henceforth DF) relaying. Alternatively, the relay might also amplify the received signal (with a power constraint) and then transmit, called amplify and forward (henceforth AF) relaying. The authors in [7] and [8] have studied the throughput characteristics of various protocols with energy harvesting relay in AF and DF relaying respectively in Rayleigh fading channels. Further relaying protocols in diverse system models have been explored in [9]–[11] for one-way relaying systems and in [12] for two-way relaying.

On the other hand, recent spectrum measurements show that although most of the spectrum band is allocated under license, the spectrum usage is very low [13], which has resulted in emerging research towards methods which facilitate spectrum sharing between users, also known as cooperative spectrum sharing. This idea of cooperative spectrum sharing has been widely pursued in literature. The authors in [14] have proposed a scheme where a secondary user co-exists with a primary user by acting as an altruistic relay and carries its transmission along with the transmission of a licensed primary user. Extension to this work with an aim to improve diversity has been studied in [15], wherein the secondary user is equipped with multiple antennas thus improving the coverage and performance of both primary and secondary user. The combination of cognitive spectrum sharing and opportunistic energy harvesting has been studied in [16], where the authors have derived maximum secondary network throughput under given outage probability constraints. Outage patterns in relay assisted cognitive user which harvests energy from primary user is studied in [17], where an underlay [18] cognitive network is studied. The transmission of the secondary user in [17] is therefore limited by the peak permissible interference power with respect to the primary transmission. In this paper we consider an overlay system paradigm, where the primary user throughput is also a concern and the secondary user helps in relaying the primary signal in exchange for spectrum access. Our main contributions and insights in this paper are listed as follows.

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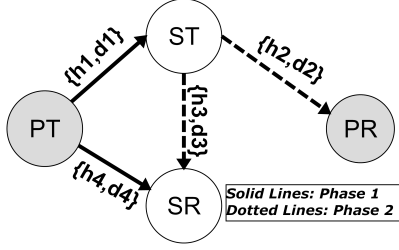


Fig. 1: Geometric model of the scheme

A. Contributions

- We propose time splitting and power sharing based cooperative spectrum sharing protocols in which an energy constrained secondary (a.k.a cognitive) user acts as a relay for the primary signal. These novel protocols address both the problems of spectrum sharing and energy harvesting, wherein the secondary user harvests energy from the primary signal, relays the primary signal, and receives spectrum access for its own transmission.
- The outage probabilities under both the protocols are evaluated for both the primary and the secondary users.
- The AF and DF relaying schemes have been separately considered while calculating the outage expressions. Note that both these schemes are fundamentally different based on the nature of the relaying used, as explained in [19]. AF relaying nodes are relatively simpler to build and are preferred over DF if privacy and security of the primary data is an issue.
- The analytical results are verified using numerical simulations and the effect of the various system parameters on the outage have been studied. The optimum value of the splitting parameters and the node placement, which give the minimum outage at the destinations, have been evaluated. From the obtained results, it is shown that the proposed PS-CSS protocol, which is based on power sharing receiving at ST, outperforms TS-PSS protocol, which employs time-splitting based receiving over a wide range of Signal to noise ratio (SNR) values.

B. Organization

The paper is organized as follows. In section II the geometry of the model and the key assumptions in this paper are explained. In sections III and IV, the TS-CSS and PS-CSS protocols are explained respectively and the expressions of the outage probabilities in both the protocols are derived within AF and DF relaying. In section V, the proposed analytical expressions are verified with simulations and the effects of various system parameters in the outage calculation are summarized. Section VI finally concludes the paper and summarizes the results.

II. SYSTEM MODEL AND GEOMETRY

The geometric model of the scheme is shown in Fig.1. Our model contains a primary transmitter and receiver (together called primary user) and a secondary transmitter and receiver (together called secondary user or cognitive user), where

we study the transmission over a block time T . The total transmission is divided into Phases 1 and 2. In Phase 1 (solid lines in Fig.1), the Primary Transmitter (PT) broadcasts its signal $x_p(\mathbb{E}[|x_p|^2] = 1)$ with a transmit power P_p . The secondary transmitter (ST) receives the signal, harvests energy, and processes the information from the signal. The Phase 2 (dotted lines in Fig.1) is further subdivided into Phase 2.a and Phase 2.b. In Phase 2.a, ST uses the harvested energy acting as an altruistic relay and forwards the primary signal to primary receiver (PR) and in Phase 2.b, ST transmits its own signal $x_s(\mathbb{E}[|x_s|^2] = 1)$ towards secondary receiver (SR). The two signals sent by ST in Phase 2 are split in time, to prevent any linear inter-combination of signals as in [14], [15] and hence avoid interference at the receivers. So, the performance constraint in these protocols is only due to channel fading and additive noise. The performance metric is taken as the outage probabilities at the receivers. Specifically, we take R_p and R_s as the threshold rates at the primary and secondary destinations respectively and declare an outage if the achieved rates fall below these respective threshold rates, where we assume that both the transmitters always have some information to send.

For this paper, we consider all the channel links to be quasi-static block fading channels, where the channel gains are independent but need not be necessarily identically distributed (I.I.N.D.). In particular, we assume Nakagami- m fading model¹ with channel coefficients h_1, h_2, h_3 and h_4 over the channels PT-ST, ST-PR, ST-SR and PT-SR respectively. Also, we represent the respective link distances by d_1, d_2, d_3 and d_4 . Consequently, $|h_i| \sim \text{Nakagami}(m, \Omega_i)$, $i \in \{1, 2, \dots, 4\}$ and $\Omega_i = d_i^{-v}$, where v is the path-loss exponent factor and m is the shape parameter of Nakagami distribution. The random variables $|h_i|^2$ represented by γ_i would then be Gamma distributed². So, $\gamma_i \sim \text{Gamma}(m, \theta_i)$, where $\theta_i = \Omega_i/m$. The SNR is defined for a signal $x(s)$ as

$$\text{SNR} = \frac{\mathbb{E}[|\text{signal component}|^2]}{\mathbb{E}[|\text{noise component}|^2]} \quad (1)$$

In DF relaying scheme, if ST fails to decode the primary signal, it is assumed that it keeps quiet in Phase 2, without transmitting its own signal [14]. In AF relaying, however, ST would go on with both ST-PR and ST-SR transmission in Phase 2, irrespective of the transmission in Phase 1.

A. Assumptions and Notation

- 1) For ease of analysis, we assume that there is no direct link between the nodes PT-PR for direct transmission (DT), so the primary user completely relies on the secondary user for transmission, which is justified in the case of urban areas or nodes placed far apart. This assumption is also in line with previous research in this area such as in [7].

¹ $(f_{|h_i|}(z) \sim \frac{2m^m}{\Gamma(m)\Omega_i^m} z^{2m-1} e^{-\frac{mz}{\Omega_i}})$

² $(f_{\gamma_i}(x) \sim \frac{x^{m-1} e^{-\frac{x}{\theta_i}}}{\Gamma(m)\theta_i^m})$

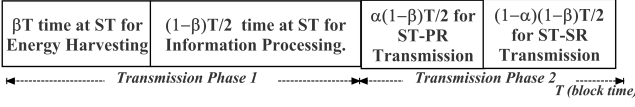


Fig. 2: Transmission in TS-CSS scheme.

- 2) We assume additive white Gaussian noise (AWGN) at all the receivers. The noise on channel link i in transmission phase j is represented as n_{ij} , and

$$n_{ij} \sim \mathcal{CN}(0, \sigma_{ij}^2) \quad i \in \{1, 2, \dots, 4\} \text{ and } j \in \{1, 2\} \quad (2)$$

where σ_{ij}^2 denotes the AWGN noise variance.

- 3) We perform the analysis assuming that the nodes are equipped with single antenna, which is a reasonable assumption in small-sized sensor nodes [1]. We further assume harvest-store-use system of energy harvesting relaying, where the energy harvested is directly and continuously used by the relaying node (secondary transmitter in this case) and the excess energy is stored for future use, as explained in [20]. We also assume that the node is equipped with an ideal storage unit with infinite battery capacity. The energy transfer efficiency however need not be 100% and the practical limit achieved is around 55% [21].
- 4) We assume that the Channel State Information (CSI) is perfectly available at the receivers. For ST, we assume that the CSI is available at the receiver side but not on the transmitting side, which is in line with previous works in this area such as in [22]–[24].

III. THE TS-CSS SCHEME

The Time Splitting - Cooperative Spectrum Sharing (TS-CSS) protocol is explained in Fig.2, where T is assumed to be the block time. In this protocol, PT harvests energy for βT of the time, and the remaining $(1-\beta)T$ time is divided equally into information processing at ST and message transmission from ST to PR and SR i.e., $(1-\beta)T/2$ time is used for information processing and $(1-\beta)T/2$ time is used in message transmission to PR and SR from ST. Further, out of the allocated $(1-\beta)T/2$ time for transmission, α of the fraction is used for ST-PR transmission and the remaining $(1-\alpha)$ for ST-SR transmission. Note that there is a two-fold time splitting, so there are two independent parameters, α and β .

A. DF Relaying scheme

The algorithm for the proposed TS-CSS protocol with DF relaying has been summarized in **Algorithm 1**. The signal received at ST from PT in Phase 1 is given by

$$y_{ST} = \sqrt{P_p} h_1 x_p + n_{11} \quad (3)$$

Since $\mathbb{E}[|x_p|^2] = 1$ and $|h_1|^2 = \gamma_1$, the Signal to Noise (SNR) at ST can be given by:

$$\text{SNR}_{ST}^{\text{TS-DF}} = \frac{P_p \gamma_1}{\sigma_{11}^2} \quad (4)$$

Algorithm 1 DF Relaying in TS-CSS protocol

▷ Phase 1

- 1: PT transmits x_p towards ST
- 2: ST harvests energy from x_p for βT time.
- 3: ST attempts to decode x_p for $(1-\beta)T/2$ time.

▷ Phase 2

- 4: **if** $R_{ST}^{\text{TS-DF}} > R_p$ **then**
- 5: ST transmits x_p for $\alpha(1-\beta)T/2$ time.
- 6: ST transmits x_s for $(1-\alpha)(1-\beta)T/2$ time.
- 7: **else**
- 8: ST keeps quiet for $(1-\beta)T/2$ time.
- 9: **end if**

From (4), the achievable rate between PT and ST can thus be calculated as

$$R_{ST}^{\text{TS-DF}} = \frac{(1-\beta)T}{2} \log_2(1 + \text{SNR}_{ST}^{\text{TS-DF}})$$

where the factor $(1-\beta)T/2$ denotes the time for which ST decodes the data, out of the block time T . Since βT time is used for energy harvesting, the harvested energy in this protocol is given by

$$E_h = \eta P_p \gamma_1 (\beta T) \quad (5)$$

where η is the energy harvesting efficiency. Next, we denote the power allocated for ST-PR transmission by P_{r_1} and that allocated for ST-SR transmission by P_{r_2} . The power extracted in any time ΔT from energy E is given by $E/\Delta T$. Therefore,

$$P_{r_1} = \frac{E_h}{\alpha(1-\beta)T/2} = \frac{2\eta P_p \beta}{\alpha(1-\beta)} \gamma_1 \quad (6a)$$

$$P_{r_2} = \frac{E_h}{(1-\alpha)(1-\beta)T/2} = \frac{2\eta P_p \beta}{(1-\alpha)(1-\beta)} \gamma_1 \quad (6b)$$

In DF relaying, ST first attempts to decode the received primary signal and then transmits it towards PR in phase 2. It transmits the primary signal for $\alpha(1-\beta)T/2$ time and then the secondary signal for $(1-\alpha)(1-\beta)T/2$ time, which are received by PR and SR respectively as

$$\begin{aligned} y_{PR} &= \sqrt{P_{r_1}} h_2 x_p + n_{22} \\ y_{SR} &= \sqrt{P_{r_2}} h_3 x_s + n_{32} \end{aligned} \quad (7)$$

Since $\mathbb{E}[|x_s|^2] = 1$, the SNR at PR and SR can be given as

$$\text{SNR}_{PR}^{\text{TS-DF}} = \frac{P_{r_1} \gamma_2}{\sigma_{22}^2} \text{ and } \text{SNR}_{SR}^{\text{TS-DF}} = \frac{P_{r_2} \gamma_3}{\sigma_{32}^2} \quad (8)$$

After substituting P_{r_1} and P_{r_2} from (6a) and (6b) respectively, we get

$$\begin{aligned} \text{SNR}_{PR}^{\text{TS-DF}} &= \frac{2\eta P_p \beta}{\alpha(1-\beta)\sigma_{22}^2} \gamma_1 \gamma_2 \\ \text{SNR}_{SR}^{\text{TS-DF}} &= \frac{2\eta P_p \beta}{(1-\alpha)(1-\beta)\sigma_{32}^2} \gamma_1 \gamma_3. \end{aligned} \quad (9)$$

Hence from (9), the achievable rate at PR and SR can thus be calculated as

$$R_{PR}^{\text{TS-DF}} = \frac{\alpha(1-\beta)T}{2} \log_2(1 + \text{SNR}_{PR}^{\text{TS-DF}}) \quad (10a)$$

$$R_{SR}^{\text{TS-DF}} = \frac{(1-\alpha)(1-\beta)T}{2} \log_2(1 + \text{SNR}_{SR}^{\text{TS-DF}}) \quad (10b)$$

Proposition 1. The outage probabilities of the Primary user, $\Pr_{out_1}^{TS-DF}$ and that of the Secondary user, $\Pr_{out_2}^{TS-DF}$ in DF relaying mode for TS-CSS protocol are given in closed form by

$$\Pr_{out_1}^{TS-DF} = 1 - [(1 - \Gamma(m, Y1)) \left(1 - \frac{1}{\Gamma(m)^2} G_{1,3}^{2,1} \left(\frac{Z1}{\theta_1 \theta_2} \middle| \mu_1, m, 0 \right) \right)] \quad (11a)$$

$$\Pr_{out_2}^{TS-DF} = 1 - [(1 - \Gamma(m, Y1)) \left(1 - \frac{1}{\Gamma(m)^2} G_{1,3}^{2,1} \left(\frac{Z2}{\theta_1 \theta_3} \middle| \mu_1, m, 0 \right) \right)] \quad (11b)$$

where,

$$Y1 = \frac{\psi_1 \sigma_{11}^2}{P_p \theta_1} \quad (12a) \quad \psi_1 = 2^{\frac{2R_p}{(1-\beta)T}} - 1 \quad (13a)$$

$$Z1 = \frac{\alpha(1-\beta)\sigma_{22}^2\psi_2}{2\eta P_p \beta} \quad (12b) \quad \psi_2 = 2^{\frac{2R_p}{\alpha(1-\beta)T}} - 1 \quad (13b)$$

$$Z2 = \frac{(1-\alpha)(1-\beta)\sigma_{32}^2\psi_3}{2\eta P_p \beta} \quad (12c) \quad \psi_3 = 2^{\frac{2R_s}{(1-\alpha)(1-\beta)T}} - 1 \quad (13c)$$

The value of $\mu_1=1$ and the function $\Gamma(\cdot, \cdot)$ is the normalized lower incomplete Gamma function³. The Meijer-G function $G_{p,q}^{m,n}$, defined in [25, eq.9.301], is given below.

$$G_{p,q}^{m,n} \left(z \middle| \begin{matrix} a_1, \dots, a_n, a_{n+1}, \dots, a_p \\ b_1, \dots, b_m, b_{m+1}, \dots, b_q \end{matrix} \right) = \frac{1}{2\pi i} \int \frac{\prod_{j=1}^n \Gamma(1 - a_j + s) \cdot \prod_{j=1}^m \Gamma(b_j - s)}{\prod_{j=n+1}^p \Gamma(a_j - s) \prod_{j=m+1}^q \Gamma(1 - b_j + s)} z^s ds \quad (14)$$

Proof: Proof of the proposition given in Appendix A. ■

B. AF relaying scheme

Algorithm 2 AF Relaying in TS-CSS protocol

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- ▷ Phase 1
 - 1: PT transmits x_p towards ST
 - 2: ST harvests energy from x_p for βT time.
 - 3: ST processes x_p for $(1 - \beta)T/2$ time.
 - ▷ Phase 2
 - 4: ST amplifies and transmits x_p for $\alpha(1 - \beta)T/2$ time.
 - 5: ST transmits x_s for $(1 - \alpha)(1 - \beta)T/2$ time.
-

This scheme is explained in **Algorithm 2**. Contrary to DF relaying, in AF relaying mode ST amplifies the received signal from PT in Phase 1, and then transmits the amplified signal to PR. The signal received at PR is given by

$$y_{PR} = \frac{\sqrt{P_{r1}}}{\sqrt{P_p \gamma_1 + \sigma_{11}^2}} y_{ST} h_2 + n_{22}. \quad (15)$$

where the term $\sqrt{P_p \gamma_1 + \sigma_{11}^2}$ in the denominator is the power constraint factor at the relay [10] chosen to ensure that the transmit power averaged over noise and interference is P_{r1} ,

$$^3 \Gamma(m, x) = \frac{\int_0^x t^{m-1} e^{-t} dt}{\Gamma(m)}$$

which is the transmit power for ST-PR transmission, given by (6a). y_{ST} is given by (3). Substituting y_{ST} in (15), we get

$$y_{PR} = \underbrace{\frac{\sqrt{P_{r1} P_p}}{\sqrt{P_p \gamma_1 + \sigma_{11}^2}} h_1 h_2 x_p}_{\text{signal component}} + \underbrace{\frac{\sqrt{P_{r1}}}{\sqrt{P_p \gamma_1 + \sigma_{11}^2}} n_{11} h_2 + n_{22}}_{\text{noise component}} \quad (16)$$

Substituting for P_{r1} and rearranging the terms, we arrive at the expression for SNR at PR as

$$\text{SNR}_{PR}^{TS-AF} = \frac{\left(\frac{2\eta P_p \beta}{\alpha(1-\beta)\sigma_{22}^2} \gamma_1 \gamma_2 \right) \cdot \left(\frac{P_p}{\sigma_{11}^2} \gamma_1 \right)}{\left(\frac{2\eta P_p \beta}{\alpha(1-\beta)\sigma_{22}^2} \gamma_1 \gamma_2 \right) + \left(\frac{P_p}{\sigma_{11}^2} \gamma_1 \right) + 1} \quad (17)$$

Using (17), the achievable rate at PR can thus be given by

$$R_{PR}^{TS-AF} = \frac{\alpha(1-\beta)T}{2} \log_2(1 + \text{SNR}_{PR}^{TS-AF}) \quad (18)$$

The secondary transmission at ST towards SR, however, is independent of the mode of relaying used and would follow the same pattern as in DF relaying. Therefore, the signal received at SR, y_{SR} , is given by (7) and the SNR at SR can be given as

$$\text{SNR}_{SR}^{TS-AF} = \frac{2\eta P_p \beta}{(1-\alpha)(1-\beta)\sigma_{32}^2} \gamma_1 \gamma_3. \quad (19)$$

and the rate at SR as

$$R_{SR}^{TS-AF} = \frac{(1-\alpha)(1-\beta)T}{2} \log_2(1 + \text{SNR}_{SR}^{TS-AF}) \quad (20)$$

We note that since there will always be transmission in Phase 2 of AF relaying scheme, it makes better utilization of the block time T.

Proposition 2. The outage probabilities of the Primary user, $\Pr_{out_1}^{TS-AF}$ and that of the Secondary user, $\Pr_{out_2}^{TS-AF}$ in AF relaying mode in TS-CSS protocol are given by

$$\Pr_{out_1}^{TS-AF} = 1 - \int_{Y1}^{\infty} \frac{y^{(m-1)} e^{-y}}{\Gamma(m)} \Gamma_u \left(m, \frac{\psi_1 (b\theta_1 y + 1)}{\theta_1 \theta_2 (ab\theta_1 y^2 - \psi_1 a y)} \right) dy \quad (21a)$$

$$\Pr_{out_2}^{TS-AF} = \frac{1}{\Gamma(m)^2} G_{1,3}^{2,1} \left(\frac{Z2}{\theta_1 \theta_3} \middle| \mu_1, m, 0 \right) \quad (21b)$$

where,

$$a = \frac{2\eta P_p \beta}{\alpha(1-\beta)\sigma_{22}^2} \quad (22a) \quad b = \frac{P_p}{\sigma_{11}^2} \quad (23a)$$

$$Y1 = \frac{\psi_1 \sigma_{11}^2}{P_p \theta_1} \quad (22b) \quad \psi_1 = 2^{\frac{2R_p}{\alpha(1-\beta)T}} - 1 \quad (23b)$$

$$Z2 = \frac{(1-\alpha)(1-\beta)\sigma_{32}^2\psi_2}{2\eta P_p \beta} \quad (22c) \quad \psi_2 = 2^{\frac{2R_s}{(1-\alpha)(1-\beta)T}} - 1 \quad (23c)$$

where the value of $\mu_1 = 1$, $\Gamma_u(\cdot, \cdot)$ is the upper normalized incomplete Gamma function⁴ and Meijer-G function is defined in (14). The integration in (21a) cannot be further simplified. It can be evaluated numerically and is studied in the simulation results.

Proof: Proof of the proposition given in Appendix B. ■

$$^4 \Gamma_u(m, x) + \Gamma(m, x) = 1$$

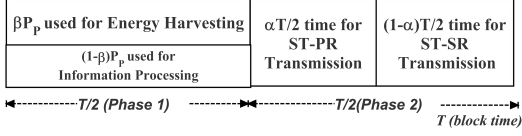


Fig. 3: Transmission in PS-CSS scheme

IV. THE PS-CSS SCHEME

The PS-CSS protocol is explained in Fig.3, where T is assumed to be the block time. In Phase 1, for $T/2$ of the time, the primary transmitter uses β of the power from the received signal (βP_p) for energy harvesting and $(1-\beta)P_p$ for information decoding. In Phase 2, the next $T/2$ time is split in the ratio $\alpha : (1-\alpha)$, where ST-PR transmission is carried in the first $\alpha T/2$ and ST-SR transmission in the remaining $(1-\alpha)T/2$ time. Note that again, the signals are split in time in Phase 2 to avoid any interference at the receivers.

A. DF Relaying Scheme

Algorithm 3 DF Relaying in PS-CSS protocol

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- ▷ Phase 1
- 1: PT transmits x_p towards ST with power P_p
 - 2: ST harvests energy from βP_p power and decodes signal with $(1-\beta)P_p$ power for $T/2$ time.
- ▷ Phase 2
- 3: **if** $R_{ST}^{PS-DF} > R_p$ **then**
 - 4: ST transmits x_p for $\alpha T/2$ time.
 - 5: ST transmits x_s for $(1-\alpha)T/2$ time.
 - 6: **else**
 - 7: ST keeps quiet for $T/2$ time.
 - 8: **end if**
-

DF relaying scheme for PS-CSS protocol has been summarized in **Algorithm 3**. The signal received at ST from PT in Phase 1 is given by (3). The ST then splits the signal between the energy harvester and information decoder. The signal received at the information receiver is given by

$$y'_{ST} = \sqrt{(1-\beta)} y_{ST} = \sqrt{(1-\beta)P_p} h_1 x_p + n_{11} \quad (24)$$

We have assumed that the noise factor is not affected by the power sharing, which is generally the case and justified in [26]. The SNR at ST is given from this equation by:

$$\text{SNR}_{ST}^{PS-DF} = \frac{(1-\beta)P_p \gamma_1}{\sigma_{11}^2}. \quad (25)$$

So, the rate at ST, R_{ST}^{PS-DF} for PS-CSS protocol in DF relaying mode can be given as

$$R_{ST}^{PS-DF} = \frac{T}{2} \log_2(1 + \text{SNR}_{ST}^{PS-DF}) \quad (26)$$

Now, the energy harvested in time $T/2$ from power βP_p at ST is given by $E_h = \eta \beta P_p \gamma_1 (T/2)$. Next, we denote the power allocated in PS-CSS protocol for ST-PR transmission by P'_{r1}

and that allocated for ST-SR transmission by P'_{r2} . Therefore,

$$\begin{aligned} P'_{r1} &= \frac{E_h}{\alpha T/2} = \frac{\eta \beta P_p}{\alpha} \gamma_1 \\ P'_{r2} &= \frac{E_h}{(1-\alpha)T/2} = \frac{\eta \beta P_p}{(1-\alpha)} \gamma_1 \end{aligned} \quad (27)$$

Like in TS-CSS protocol, ST first transmits the signal x_p with power P'_{r1} which is received by PR and then transmits its own signal x_s with power P'_{r2} , which is received by SR. Therefore, the signals received at PR and SR in phase 2 are given by

$$\begin{aligned} y_{PR} &= \sqrt{P'_{r1}} h_2 x_p + n_{22} \\ y_{SR} &= \sqrt{P'_{r2}} h_3 x_s + n_{32} \end{aligned} \quad (28)$$

The SNR at PR and SR, after substituting P'_{r1} and P'_{r2} from (27) are given by

$$\begin{aligned} \text{SNR}_{PR}^{PS-DF} &= \frac{\eta P_p \beta}{\alpha \sigma_{22}^2} \gamma_1 \gamma_2 \\ \text{SNR}_{SR}^{PS-DF} &= \frac{\eta \beta P_p}{(1-\alpha) \sigma_{32}^2} \gamma_1 \gamma_3 \end{aligned} \quad (29)$$

The corresponding rates at PR and SR can be given from (29) as

$$\begin{aligned} R_{PR}^{PS-DF} &= \frac{\alpha T}{2} \log_2(1 + \text{SNR}_{PR}^{PS-DF}) \\ R_{SR}^{PS-DF} &= \frac{(1-\alpha)T}{2} \log_2(1 + \text{SNR}_{SR}^{PS-DF}) \end{aligned} \quad (30)$$

Proposition 3. The outage probabilities of the Primary user, Pr_{out1}^{PS-DF} and that of the secondary user, Pr_{out2}^{PS-DF} in DF relaying mode in PS-CSS protocol are given in closed form by

$$\begin{aligned} \text{Pr}_{out1}^{PS-DF} &= 1 - [(1 - \Gamma(m, Y1)) \\ &\quad \left(1 - \frac{1}{\Gamma(m)^2} G_{1,3}^{2,1} \left(\frac{Z1}{\theta_1 \theta_2} \middle| \begin{matrix} \mu_1 \\ m, m, 0 \end{matrix} \right) \right)] \end{aligned} \quad (31a)$$

$$\begin{aligned} \text{Pr}_{out2}^{PS-DF} &= 1 - [(1 - \Gamma(m, Y1)) \\ &\quad \left(1 - \frac{1}{\Gamma(m)^2} G_{1,3}^{2,1} \left(\frac{Z2}{\theta_1 \theta_3} \middle| \begin{matrix} \mu_1 \\ m, m, 0 \end{matrix} \right) \right)] \end{aligned} \quad (31b)$$

where,

$$Y1 = \frac{\psi_1 \sigma_{11}^2}{(1-\beta)P_p \theta_1} \quad (32a) \quad \psi_1 = 2^{\frac{2R_p}{T}} - 1 \quad (33a)$$

$$Z1 = \frac{\alpha \sigma_{22}^2 \psi_2}{\eta P_p \beta} \quad (32b) \quad \psi_2 = 2^{\frac{2R_p}{\alpha T}} - 1 \quad (33b)$$

$$Z2 = \frac{(1-\alpha) \psi_3 \sigma_{32}^2}{\eta P_p \beta} \quad (32c) \quad \psi_3 = 2^{\frac{2R_s}{(1-\alpha)T}} - 1 \quad (33c)$$

The value of $\mu_1 = 1$; the lower incomplete Gamma function, $\Gamma(m, x)$ and the Meijer-G function, $G_{p,q}^{m,n}$ are as defined in (14).

Proof: Proof of the Proposition given in Appendix C. Note that although the forms of outage might look same in both TS-CSS and PS-CSS scheme, that is not the case as the variables $Y1, Z1 \dots$ are different in each context. ■

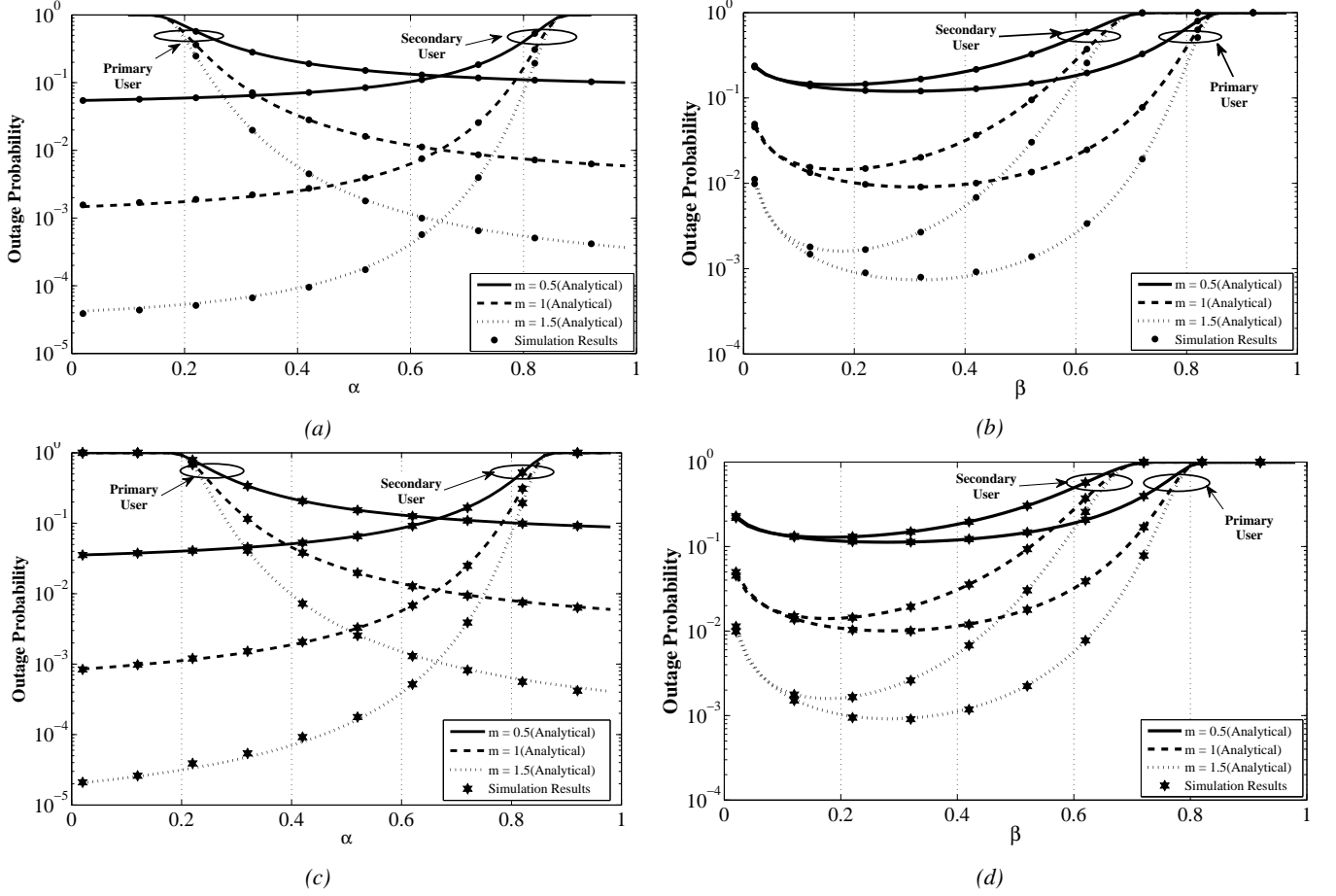


Fig. 4: Variation patterns of outage probability for primary and secondary users in TS-CSS protocol. The variation of outage in DF relaying w.r.t α have been shown in Fig.4a, and Fig.4b shows the plot versus β . The respective variations in AF relaying have been plotted in figures 4c and 4d. The other parameters are $\eta = 1$, $d_{PT-ST} = 1$ and $P_p/\sigma^2 = 40\text{dB}$.

Algorithm 4 AF Relaying in PS-CSS protocol

▷ Phase 1

- 1: PT transmits x_p towards ST
- 2: ST harvests energy from βP_p power and decodes signal with $(1 - \beta)P_p$ power for $T/2$ time.

▷ Phase 2

- 3: ST amplifies and transmits x_p for $\alpha T/2$ time.
- 4: ST transmits x_s for $(1 - \alpha)T/2$ time.

B. AF Relaying Scheme

The AF relaying mode for PS-CSS protocol has been summarized in **Algorithm 4**. In AF relaying mode, the signal received at ST in Phase 1, y'_{ST} , is given by (24). In Phase 2, the primary signal at ST is directly forwarded to PR for $\alpha T/2$ time, and then ST transmits its own signal towards SR for the remaining slot time of $(1 - \alpha)T/2$. Therefore, the signal received at PR is given by

$$y_{PR} = \frac{\sqrt{P'_{r1}}}{\sqrt{(1 - \beta)P_p\gamma_1 + \sigma_{11}^2}} y'_{ST} h_2 + n_{22}. \quad (34)$$

where the factor $\sqrt{(1 - \beta)P_p\gamma_1 + \sigma_{11}^2}$ in the denominator is the power constraint factor and P'_{r1} is given in (27).

Substituting y'_{ST} from (24), we get

$$y_{PR} = \underbrace{\frac{\sqrt{P'_{r1} P_p (1 - \beta)}}{\sqrt{(1 - \beta)P_p\gamma_1 + \sigma_{11}^2}} h_1 h_2}_{\text{Signal Component}} + \underbrace{\frac{\sqrt{P'_{r1}}}{\sqrt{(1 - \beta)P_p\gamma_1 + \sigma_{11}^2}} h_2 n_{11} + n_{22}}_{\text{Noise Component}}. \quad (35)$$

Substituting the value of P'_{r1} and rearranging the terms, we arrive at the result for SNR at PR as

$$\text{SNR}_{PR}^{\text{PS-AF}} = \frac{\left(\frac{\eta \beta P_p}{\alpha \sigma_{22}^2} \gamma_1 \gamma_2 \right) \cdot \left(\frac{(1 - \beta) P_p}{\sigma_{11}^2} \gamma_1 \right)}{\left(\frac{\eta \beta P_p}{\alpha \sigma_{22}^2} \gamma_1 \gamma_2 \right) + \left(\frac{(1 - \beta) P_p}{\sigma_{11}^2} \gamma_1 \right) + 1} \quad (36)$$

Using (36), the achievable rate at PR in AF mode would be

$$R_{PR}^{\text{PS-AF}} = \frac{\alpha T}{2} \log_2(1 + \text{SNR}_{PR}^{\text{PS-AF}}) \quad (37)$$

The transmission towards SR in Phase 2 is independent of the mode of relaying used at ST. So, the signal received at SR,

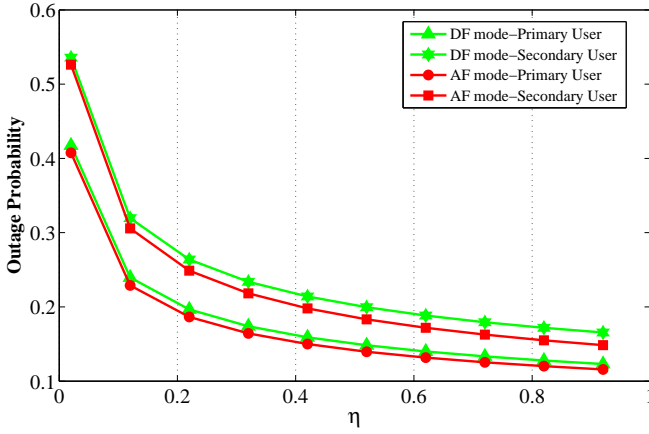


Fig. 5: Outage probability plot versus η for $\alpha = 0.7$ and $\beta = 0.3$ for both primary and secondary users in TS-CSS Protocol, $m=1$.

y_{SR} , is given by (28). The SNR at SR is given by

$$\text{SNR}_{\text{SR}}^{\text{PS-AF}} = \frac{\eta\beta P_p}{(1-\alpha)\sigma_{32}^2} \gamma_1 \gamma_3 \quad (38)$$

As was the case with TS-CSS protocol, there would always be transmission in Phase 2 in AF relaying scheme, hence making better utilization of the block time T . The achieved rate at SR would thus be given by

$$R_{\text{SR}}^{\text{PS-AF}} = \frac{(1-\alpha)T}{2} \log_2(1 + \text{SNR}_{\text{SR}}^{\text{PS-AF}}) \quad (39)$$

Proposition 4. The outage probabilities of the Primary user, $\text{Pr}_{\text{out}_1}^{\text{PS-AF}}$ and that of the secondary user, $\text{Pr}_{\text{out}_2}^{\text{PS-AF}}$ in AF relaying mode in PS-CSS protocol are given by

$$\text{Pr}_{\text{out}_1}^{\text{PS-AF}} = 1 - \int_{Y_1}^{\infty} \frac{y^{(m-1)} e^{-y}}{\Gamma(m)} \Gamma_u \left(m, \frac{\psi_1(b\theta_1 y + 1)}{\theta_1 \theta_2 (ab\theta_1 y^2 - \psi_1 a y)} \right) dy \quad (40a)$$

$$\text{Pr}_{\text{out}_2}^{\text{PS-AF}} = \frac{1}{\Gamma(m)^2} G_{1,3}^{2,1} \left(\frac{Z2}{\theta_1 \theta_3} \middle| \begin{matrix} \mu_1 \\ m, m, 0 \end{matrix} \right) \quad (40b)$$

where,

$$a = \frac{\eta P_p \beta}{\alpha \sigma_{22}^2} \quad (41a)$$

$$Y_1 = \frac{\psi_1 \sigma_{11}^2}{P_p \theta_1} \quad (41b)$$

$$Z2 = \frac{(1-\alpha)\psi_2 \sigma_{32}^2}{\eta P_p \beta} \quad (41c)$$

$$b = \frac{(1-\beta)P_p}{\sigma_{11}^2} \quad (42a)$$

$$\psi_1 = 2^{\frac{2R_p}{\alpha T}} - 1 \quad (42b)$$

$$\psi_2 = 2^{\frac{2R_s}{(1-\alpha)T}} - 1 \quad (42c)$$

The value of $\mu_1 = 1$, $\Gamma_u(\cdot, \cdot)$ is the upper normalized incomplete Gamma function; the Meijer-G function is defined in (14). The integration in (40a) cannot be further simplified and is studied numerically in the simulation results.

Proof: Proof of the Proposition given in Appendix D. ■

V. SIMULATION RESULTS AND DISCUSSIONS

To study the effect of various system parameters such as α, β, η and distance between the nodes, the analytical results are plotted along with simulation results. The numerical simulations are carried by averaging over a million (10^6)

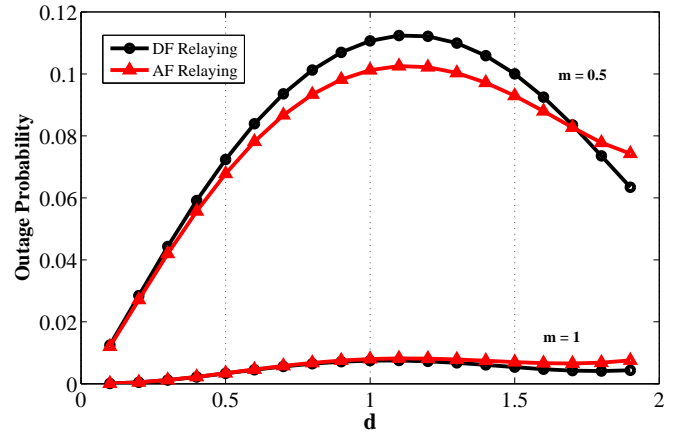


Fig. 6: Outage probability P_{out_1} (Primary User) versus d in TS-CSS Protocol. The distance $d_{PT-ST} = d$ and $d_{ST-PR} = 2 - d$.

independent random realizations of the Nakagami- m fading channels h_1, h_2, h_3 and h_4 . The results for each protocol are plotted separately and various modes and protocols are compared with each other in terms of outage. Unless otherwise specified, the AWGN noise variance is taken to be equal along all channels, and the SNR $\frac{P_p}{\sigma^2} = 40$ dB. The path loss exponent $\nu=3$ which is true in cases concerning urban areas ([7], [27]). The nodes are placed such that the distances between the links $PT - ST = ST - PR = 1\text{m}$ and $ST - SR = PT - SR = 0.5\text{m}$ as justified by [15], [17]. The energy harvesting efficiency(η) is taken to be 1, and the threshold rates $R_p=R_s=1$ bit/sec/Hz. The optimal values for α ($0 < \alpha < 1$) and β ($0 < \beta < 1$) are those values, which minimize the system outage of primary and secondary systems. We note that the obtained results only show performance upper bounds, as many factors like packet loss, errors and practically achieved values of parameters which would further decrease the throughput are not considered here.

A. TS-CSS Relaying Scheme

Fig.4a shows the variation of outage probability with respect to the time splitting ratio α for a value of $\beta = 0.3$ in DF relaying mode. Clearly the value of α at which the outages of primary and secondary are equal is independent of the value of m , the Nakagami scale parameter, and is equal to 0.65. This value can be derived analytically too by equating (11a) and (11b). The variation patterns of outage with respect to β are given in Fig.4b, and the value of β for which minimum outage is observed is around 0.3 for $m=1$ and $m=1.5$ and around 0.25 for $m=0.5$. Fig.4b also suggests that after exceeding a threshold value of β around 0.75 for primary and 0.8 for secondary user, the outage goes to 1, or the throughput $\rightarrow 0$. This can be explained as follows. For the primary user, it requires certain power threshold to transmit the signal, and once that much energy is harvested, increasing β beyond this value would not give any additional advantage. Furthermore, when $\beta > 0.8$, very less time is given for information decoding compared to energy harvesting, so the former becomes the dominating factor in deciding the outage. Also, larger β implies lesser time for transmission at ST (refer Fig.2). For secondary user,

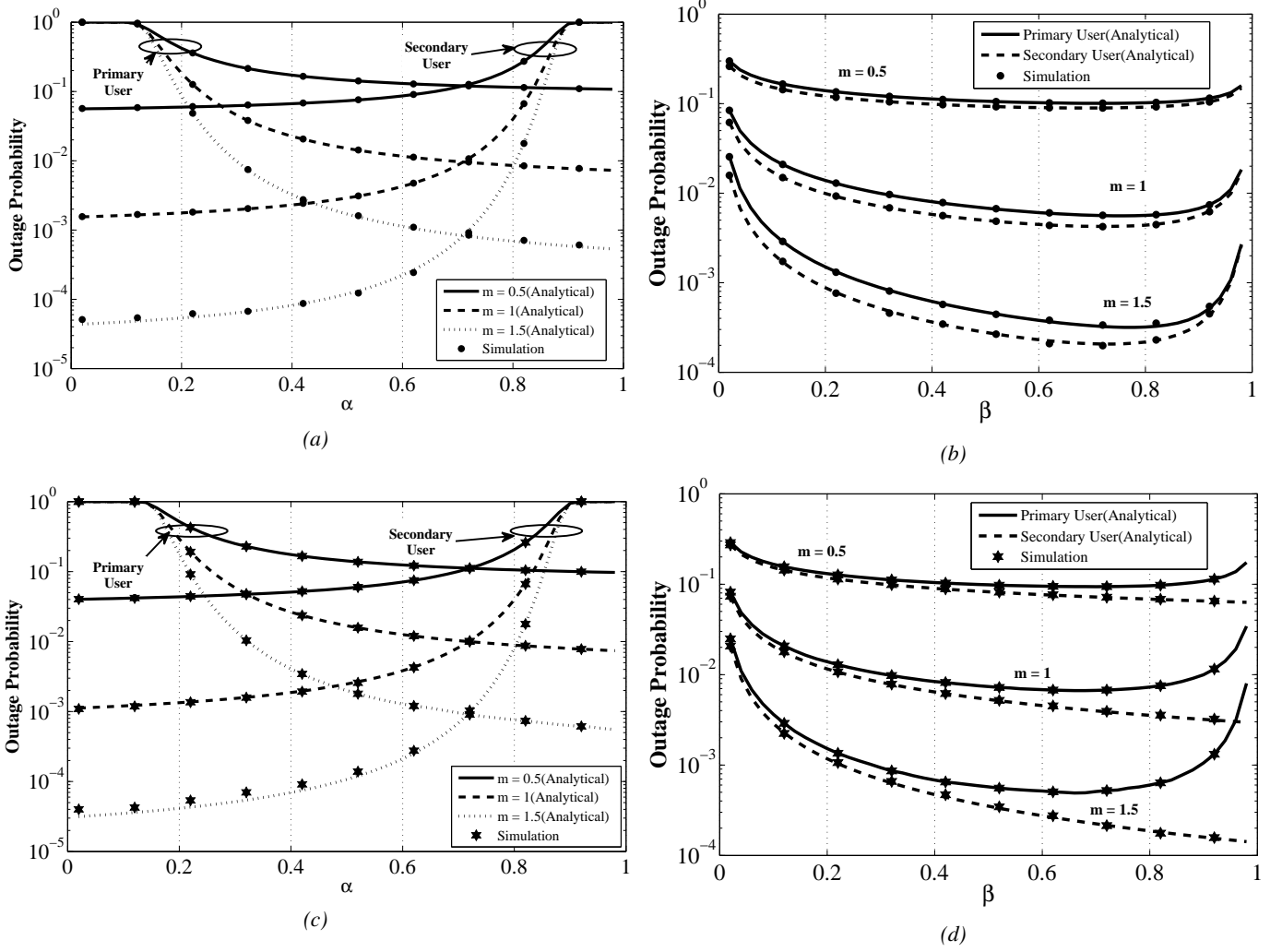


Fig. 7: Variation patterns of outage probability for primary and secondary users in PS-CSS protocol. Fig.7a shows the plot of outage versus α and Fig.7b shows the plot versus β in DF Relaying mode. The respective outage variations in AF Relaying mode are shown in figures 7c and 7d. The other parameters are $\eta = 1$, $d_{PT-ST} = 1$ and $P_p/\sigma^2 = 40\text{dB}$.

however, the time given for decoding at ST becomes the dominating factor rather earlier (at a lesser value of $\beta \approx 0.75$), because the distance between ST-SR is less than that of ST-PR, and thus it requires lesser energy threshold to transmit the signal to SR. So increasing β beyond this value would ultimately prove detrimental.

Similar trend for AF relaying can be observed in Fig.4c against α for a value of $\beta = 0.3$ and in Fig.4d against β for a value of $\alpha = 0.7$. The shapes of the plots and threshold values suggest that the overall pattern in TS-CSS relaying scheme is similar irrespective of the mode of relaying used at ST for relaying data to PR.

The variation of the outage probability with the energy harvesting efficiency η for both the protocols for $m=1$ (Rayleigh fading) is shown in Fig.5. The graph demonstrates the fact that better energy harvesting circuits would improve the system performance to a great extent. Fig.5 also establishes the marginal performance gain in AF relaying compared to DF relaying over a wide range of energy harvesting efficiency, η .

The distances between PT-ST and ST-PR are both taken to

be 1m in each of the above results. However, an interesting plot in Fig.6 is obtained by varying the position of the relaying secondary transmitter and calculating the primary user outage in TS-CSS scheme in AF and DF relaying modes, for $\alpha = 0.7$, $\beta = 0.3$ and $\eta = 1$. The outage increases as ST is moved away from PT because as the distance between PT-ST increases, the received signal strength at ST will decrease owing to a larger path loss and hence larger outages at the destination. However, when the relaying ST is close to the destination PR, the outage again decreases as the energy harvested, though very less, is sufficient to transmit the signal to PR, located near ST. So, the ideal placement of the relay, even with a cooperative spectrum sharing mechanism, is close to PT, which agrees to the result in [7]. Another conclusion evident from the graph is that when ST is near to PT than PR, AF relaying gives higher throughput than DF relaying and vice-versa when ST is near PR, although the performance gain in either case is rather marginal.

B. PS-CSS Relaying scheme

This protocol also showed similar behavior to that of the TS-CSS protocol in terms of the variation in the outage probability. Remarkably, comparison of Fig.4a and Fig.7a suggests that the variation of outage with respect to the time-splitting parameter α in both protocols follows the same pattern, for each value of m . This can be explained as follows. Both TS-CSS and PS-CSS protocol employ the parameter α as the time-sharing parameter in Phase 2, so they follow the same pattern of variation with respect to α . Moreover, this pattern is also observed in AF mode, evident from Fig.7a and Fig.7c.

On the other hand, the variation of β in Fig.7b in DF relaying does not quite follow the pattern as in Fig.4b. The optimal value of β is around 0.7 for all values of m . It is also clear that there is also no threshold β as all values produce a non-zero throughput. The reason for this is as follows. The time splitting in transmission Phase 2 of the protocol is independent of the parameter β , so a larger β would not alter the transmission time at ST (refer Fig.3) and hence has lesser dominance over the outage at both secondary and primary user. The variation of energy harvesting efficiency η follows the same pattern in both the protocols, as more efficiency in both cases leads to lesser outage probability (Fig.5).

Another interesting observation is obtained by comparing Fig.7b (DF) with Fig.7d (AF). Although the variation is identically different from their TS-CSS counterparts, the outage probability of the secondary user in amplify forward mode is a monotonically decreasing function of β , suggesting that greater the value of β chosen at ST, better will be the performance of the secondary user. This is because if β is high, ST will get greater transmit power and hence better performance. This is compared with decode forward relaying, where increasing β would in turn reduce the decoding capabilities at ST, and if ST has failed to decode the signal in Phase 1, it does not transmit its own signal too in phase 2, which is not the case with AF relaying.

Finally, both the relaying schemes are compared with each other over a wide range of SNRs and the plot is shown in Fig.8 for $m=1$ which corresponds to Rayleigh fading channel. Clearly, the PS-CSS relaying scheme outperforms the TS-CSS relaying scheme over a wide range of SNRs for both the primary and secondary users. So we conclude that the power sharing based receiving has better throughput compared to time splitting receiving in a Rayleigh fading channel.

VI. CONCLUSION

With an intention of addressing both the issues of energy constraint and spectrum efficiency in wireless networks, we propose two protocols in this paper to carry transmission in a spectrum sharing scenario between a primary user and an energy harvesting secondary user and analyze outage probabilities of these protocols. The outage expressions are plotted with respect to system parameters, and it is found out that power sharing based relaying is better than time splitting based relaying for cooperative spectrum sharing protocols over a wide range of SNRs at optimum values of α and β . It is also found that AF relaying scheme is better than DF

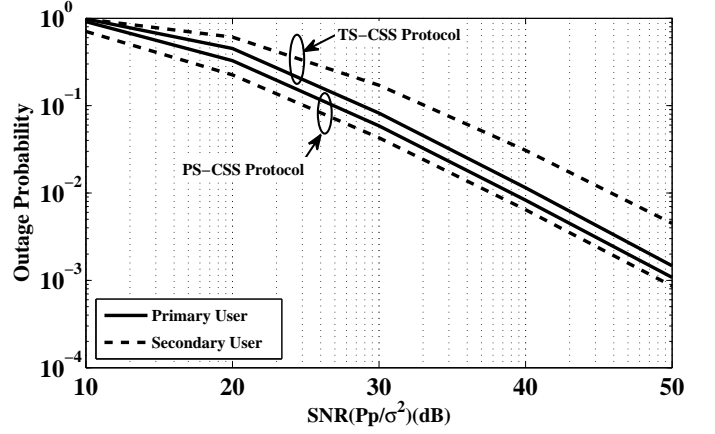


Fig. 8: Comparison of TS-CSS and PS-CSS relaying schemes for different SNR values in AF relaying. The values of $\alpha = 0.7, \beta = 0.4, \eta = 1$ and $m = 1$.

relaying for most cases except when ST is very near to PR, in which case the throughput of DF relaying has slightly better throughput. The explained protocols enables efficient usage of the spectrum without adversely effecting the performance of the primary user and promises longer lifetime of the sensor nodes through energy harvesting.

APPENDIX A

PROOF FOR PROPOSITION 1

Proof: An outage is declared for the primary user if any of the links PT-ST or ST-PR fail to achieve the threshold rate required to decode the signal (R_p). For the secondary user, outage is dependent on the links PT-ST and ST-PR, and the decoding rate at SR has to meet the threshold R_s .

$$\therefore \quad P_{r_{out1}}^{TS-DF} = 1 - [\Pr(R_{ST}^{TS-DF} > R_p) \Pr(R_{PR}^{TS-DF} > R_p)] \quad (A.1a)$$

$$P_{r_{out2}}^{TS-DF} = 1 - [\Pr(R_{ST}^{TS-DF} > R_p) \Pr(R_{SR}^{TS-DF} > R_s)] \quad (A.1b)$$

(i) $\Pr(R_{ST}^{TS-DF} > R_p)$: Using (4) for SNR_{ST}^{TS-DF} , we get

$$\begin{aligned} \Pr(R_{ST}^{TS-DF} > R_p) &= \Pr\left(SNR_{ST}^{TS-DF} > 2^{\frac{2R_p}{(1-\beta)T}} - 1\right) \\ \Rightarrow \Pr(R_{ST}^{TS-DF} > R_p) &= \Pr\left(\frac{P_p Y_1}{\sigma_{11}^2} > \psi_1\right) \\ &= \Pr\left(\gamma_1 > \frac{\psi_1 \sigma_{11}^2}{P_p}\right) \end{aligned}$$

for ψ_1 in (13a). We know from the knowledge of Gamma random variables, that

$$\begin{aligned} \Pr(\gamma_1 > X) &= 1 - \Gamma(m, \frac{X}{\theta_1}) \quad (A.2) \\ \therefore \Pr(R_{ST}^{TS-DF} > R_p) &= 1 - \Gamma(m, Y1) \end{aligned}$$

for $Y1$ in (12a).

(ii) $\Pr(R_{PR}^{TS-DF} > R_p)$: Using (10a) for R_{PR}^{TS-DF} , we can write

$$\begin{aligned} \Pr(R_{PR}^{TS-DF} > R_p) &= \Pr\left(\text{SNR}_{PR}^{TS-DF} > 2^{\frac{2R_p}{\alpha(1-\beta)T}} - 1\right) \\ &\Rightarrow \Pr(R_{PR}^{TS-DF} > R_p) = \Pr\left(\frac{2\eta P_p \beta}{\alpha(1-\beta)\sigma_{22}^2} \gamma_1 \gamma_2 > \psi_2\right) \\ &= \Pr(\gamma_1 \gamma_2 > Z_1). \end{aligned}$$

for ψ_2 in (13b) and Z_1 in (12b). Karagiannidis and Mallik in [28] have found explicit closed form expressions for CDF of product of Gamma random variables. Using their result,

$$\begin{aligned} \Pr(R_{PR}^{TS-DF} > R_p) &= \Pr(\gamma_1 \gamma_2 > Z_1) \\ &= 1 - \left(\frac{1}{\Gamma(m)^2} G_{1,3}^{2,1} \left(\frac{Z_1}{\theta_1 \theta_2} \middle| m, m, 0 \right) \right) \end{aligned} \quad (\text{A.3})$$

where $\mu_1=1$.

(iii) $\Pr(R_{SR}^{TS-DF} > R_s)$:

$$\begin{aligned} \Pr(R_{SR}^{TS-DF} > R_s) &= \Pr\left(\text{SNR}_{SR}^{TS-DF} > 2^{\frac{2R_s}{(1-\alpha)(1-\beta)T}} - 1\right) \\ &\Rightarrow \Pr(R_{SR}^{TS-DF} > R_s) = \Pr\left(\frac{2\eta P_p \beta}{(1-\alpha)(1-\beta)\sigma_{32}^2} \gamma_1 \gamma_3 > \psi_3\right) \\ &= \Pr(\gamma_1 \gamma_3 > Z_2). \end{aligned}$$

for ψ_3 in (13c) and Z_2 in (12c). Therefore,

$$\begin{aligned} \Pr(R_{SR}^{TS-DF} > R_s) &= \Pr(\gamma_1 \gamma_3 > Z_2) \\ &= 1 - \left(\frac{1}{\Gamma(m)^2} G_{1,3}^{2,1} \left(\frac{Z_2}{\theta_1 \theta_3} \middle| m, m, 0 \right) \right) \end{aligned}$$

where $\mu_1=1$. Substituting the above results in (A.1a) and (A.1b) gives the relations for \Pr_{out1}^{TS-DF} and \Pr_{out2}^{TS-DF} as in (11a) and (11b) respectively. ■

APPENDIX B

PROOF FOR PROPOSITION 2

Proof: The outage probability in this case is defined as

$$\Pr_{out1}^{TS-AF} = \Pr(R_{PR}^{TS-AF} < R_p) \quad (\text{B.1a})$$

$$\Pr_{out2}^{TS-AF} = \Pr(R_{SR}^{TS-AF} < R_s) \quad (\text{B.1b})$$

From (18), we can write $\Pr_{out1}^{TS-AF} = \Pr(R_{PR}^{TS-AF} < R_p)$ as

$$\begin{aligned} &\Pr\left(\frac{\alpha(1-\beta)T}{2} \log_2(1 + \text{SNR}_{PR}^{TS-AF}) < R_p\right) \\ &= \Pr(\text{SNR}_{PR}^{TS-AF} < 2^{\frac{2R_p}{\alpha(1-\beta)T}} - 1) \\ &= \Pr\left(\frac{\left(\frac{2\eta P_p \beta}{\alpha(1-\beta)\sigma_{22}^2} \gamma_1 \gamma_2\right) \cdot \left(\frac{P_p}{\sigma_{11}^2} \gamma_1\right)}{\left(\frac{2\eta P_p \beta}{\alpha(1-\beta)\sigma_{22}^2} \gamma_1 \gamma_2\right) + \left(\frac{P_p}{\sigma_{11}^2} \gamma_1\right) + 1} < \psi_1\right) \\ &= \Pr\left(\frac{(a\gamma_1 \gamma_2) \cdot (b\gamma_1)}{(a\gamma_1 \gamma_2) + (b\gamma_1) + 1} < \psi_1\right) \end{aligned}$$

for a, b as given in (22a) and (23a) respectively. This gives us

$$\begin{aligned} \Pr_{out1}^{TS-AF} &= \Pr\left(\frac{ab\gamma_1^2 \gamma_2}{a\gamma_1 \gamma_2 + b\gamma_1 + 1} < \psi_1\right) \\ &= \Pr(ab\gamma_1^2 \gamma_2 < a\psi_1 \gamma_1 \gamma_2 + b\psi_1 \gamma_1 + \psi_1) \\ &= \Pr(\gamma_2(ab\gamma_1^2 - a\psi_1 \gamma_1) < \psi_1(b\gamma_1 + 1)) \end{aligned}$$

which can be written as

$$\begin{aligned} \Pr_{out1}^{TS-AF} &= \Pr(R_{PR}^{TS-AF} < R_p) \\ &= \int_0^\infty f_{\gamma_1}(Z) \Pr(\gamma_2(abZ^2 - a\psi_1 Z) < \psi_1(bZ + 1)) dZ \end{aligned} \quad (\text{B.2})$$

Consider the expression $\Pr(\gamma_2(abZ^2 - a\psi_1 Z) < \psi_1(bZ + 1))$ inside the integral. Let $\rho = \frac{\psi_1}{b}$. From (B.2), if the factor $\rho > \gamma_1$, the LHS of the expression is negative and RHS is positive, so that $\Pr(-ve < +ve) = 1$. Else, if $\rho < \gamma_1$, this quantity can be written as $\Pr(\gamma_2 < \frac{\psi_1(bZ + 1)}{(abZ^2 - a\psi_1 Z)})$. So (B.2) can be written as

$$\begin{aligned} \Pr_{out1}^{TS-AF} &= \int_0^\rho \left(f_{\gamma_1}(Z) \cdot \Pr\left(\gamma_2 > \frac{\psi_1(bZ + 1)}{abZ^2 - \psi_1 aZ}\right) \right) dZ \\ &\quad + \int_\rho^\infty \left(f_{\gamma_1}(Z) \cdot \Pr\left(\gamma_2 < \frac{\psi_1(bZ + 1)}{abZ^2 - \psi_1 aZ}\right) \right) dZ \end{aligned} \quad (\text{B.3})$$

Using the knowledge of Gamma random variables reduces (B.3) to

$$\Pr_{out1}^{TS-AF} = 1 - \int_\rho^\infty \frac{Z^{m-1} e^{-Z/\theta_1}}{\Gamma(m) \theta_1^m} \Gamma_u\left(\frac{\psi_1(bZ + 1)}{\theta_2(abZ^2 - \psi_1 aZ)}\right) dZ.$$

The substitution $Z/\theta_1 = y$ gives us the desired result for \Pr_{out1}^{TS-AF} as an integration in (21a). Due to higher order terms inside the inner Gamma function, the integration cannot be further simplified and hence is solved off line numerically in the simulation results.

The outage of the secondary user is given in (B.1b) and is calculated in proof for 1. ■

APPENDIX C

PROOF FOR PROPOSITION 3

Proof: In the PS-CSS protocol too, the outage probabilities are defined as

$$\Pr_{out1}^{TS-AF} = 1 - [\Pr(R_{ST}^{TS-AF} > R_p) \Pr(R_{PR}^{TS-AF} > R_p)] \quad (\text{C.1a})$$

$$\Pr_{out2}^{TS-AF} = 1 - [\Pr(R_{ST}^{TS-AF} > R_p) \Pr(R_{SR}^{TS-AF} > R_s)] \quad (\text{C.1b})$$

We evaluate the expression term by term.

(i) $\Pr(R_{ST}^{PS-DF} > R_p)$: Using SNR_{ST}^{PS-DF} from (25), we get

$$\begin{aligned} \Pr(R_{ST}^{PS-DF} > R_p) &= \Pr\left(\frac{T}{2} \log_2(1 + \text{SNR}_{ST}^{PS-DF}) > R_p\right) \\ &= \Pr\left(\text{SNR}_{ST}^{PS-DF} > 2^{\frac{2R_p}{T}} - 1\right) \\ &\Rightarrow \Pr(R_{ST}^{PS-DF} > R_p) = \Pr\left(\frac{(1-\beta)P_p \gamma_1}{\sigma_{11}^2} > \psi_1\right) \\ &= \Pr\left(\gamma_1 > \frac{\psi_1 \sigma_{11}^2}{P_p(1-\beta)}\right) \end{aligned}$$

for ψ_1 in (33a).

$$\therefore \Pr(R_{ST}^{PS-DF} > R_p) = 1 - \Gamma(m, Y_1). \quad (\text{C.2})$$

using CDF for Gamma random variables as in A.2, and Y_1 given in 32a.

(ii) $\Pr(R_{PR}^{PS-DF} > R_p)$: Using equation for R_{PR}^{PS-DF} , we can write

$$\begin{aligned}\Pr(R_{PR}^{PS-DF} > R_p) &= \Pr\left(\frac{\alpha T}{2} \log_2(1 + \text{SNR}_{PR}^{PS-DF}) > R_p\right) \\ &= \Pr\left(\text{SNR}_{PR}^{PS-DF} > 2^{\frac{2R_p}{\alpha T}} - 1\right) \\ \Rightarrow \Pr(R_{PR}^{PS-DF} > R_p) &= \Pr\left(\frac{\eta P_p \beta}{\alpha \sigma_{22}^2} \gamma_1 \gamma_2 > \psi_2\right) \\ &= \Pr(\gamma_1 \gamma_2 > Z1).\end{aligned}$$

for ψ_2 in (33b) and $Z1$ in (32b).

$$\begin{aligned}\therefore \Pr(R_{PR}^{PS-DF} > R_p) &= \Pr(\gamma_1 \gamma_2 > Z1) \\ &= 1 - \left(\frac{1}{\Gamma(m)^2} G_{1,3}^{2,1} \left(\frac{Z1}{\theta_1 \theta_2} \middle| \mu_1 \right) \right) \quad (C.3)\end{aligned}$$

where $\mu_1=1$.

(iii) $\Pr(R_{SR}^{PS-DF} > R_s)$:

$$\begin{aligned}\Pr(R_{SR}^{PS-DF} > R_s) &= \Pr\left(\frac{(1-\alpha)T}{2} \log_2(1 + \text{SNR}_{SR}^{PS-DF}) > R_s\right) \\ &= \Pr\left(\text{SNR}_{SR}^{PS-DF} > 2^{\frac{2R_s}{(1-\alpha)T}} - 1\right) \\ \Rightarrow \Pr(R_{SR}^{PS-DF} > R_s) &= \Pr\left(\frac{\eta P_p \beta}{(1-\alpha)\sigma_{32}^2} \gamma_1 \gamma_3 > \psi_3\right) \\ &= \Pr(\gamma_1 \gamma_3 > Z2).\end{aligned}$$

for ψ_3 in (33c) and $Z2$ in (32c). Therefore,

$$\begin{aligned}\Pr(R_{SR}^{PS-DF} > R_s) &= \Pr(\gamma_1 \gamma_3 > Z2) \\ &= 1 - \left(\frac{1}{\Gamma(m)^2} G_{1,3}^{2,1} \left(\frac{Z2}{\theta_1 \theta_3} \middle| \mu_1 \right) \right) \quad (C.4)\end{aligned}$$

where $\mu_1=1$. Substituting the above results in (C.1a) and (C.1b) gives the relations for \Pr_{out1}^{PS-DF} and \Pr_{out2}^{PS-DF} as in (31a) and (31b) respectively. ■

APPENDIX D

PROOF FOR PROPOSITION 4

Proof: The outage probability in this case is defined in the same way as in AF relaying in TS-CSS protocol.

$$\Pr_{out1}^{PS-AF} = \Pr(R_{PR}^{PS-AF} < R_p) \quad (D.1a)$$

$$\Pr_{out2}^{PS-AF} = \Pr(R_{SR}^{PS-AF} < R_s) \quad (D.1b)$$

From (37), we can write $\Pr_{out1}^{PS-AF} = \Pr(R_{PR}^{PS-AF} < R_p)$ as

$$\begin{aligned}\Pr\left(\frac{\alpha T}{2} \log_2(1 + \text{SNR}_{PR}^{PS-AF}) < R_p\right) \\ &= \Pr(\text{SNR}_{PR}^{PS-AF} < 2^{\frac{2R_p}{\alpha T}} - 1) \\ &= \Pr\left(\frac{\left(\frac{\eta \beta P_p}{\alpha \sigma_{22}^2} \gamma_1 \gamma_2\right) \cdot \left(\frac{(1-\beta) P_p}{\sigma_{11}^2} \gamma_1\right)}{\left(\frac{\eta \beta P_p}{\alpha \sigma_{22}^2} \gamma_1 \gamma_2\right) + \left(\frac{(1-\beta) P_p}{\sigma_{11}^2} \gamma_1\right) + 1} < \psi_1\right) \\ &= \Pr\left(\frac{(a \gamma_1 \gamma_2) \cdot (b \gamma_1)}{(a \gamma_1 \gamma_2) + (b \gamma_1) + 1} < \psi_1\right)\end{aligned}$$

for a, b as given in (41a) and (42a) respectively. The expression is similar to that of AF relaying in TS-CSS protocol. Following the same procedure, the outage probability can be given as

$$\Pr_{out1}^{PS-AF} = 1 - \int_0^\infty \frac{Z^{m-1} e^{-Z/\theta_1}}{\Gamma(m) \theta_1^m} \Gamma_u\left(\frac{\psi_1(bZ+1)}{\theta_2(abZ^2 - \psi_1 aZ)}\right) dZ \quad (D.2)$$

The substitution $Z/\theta_1 = y$ gives us the desired result for \Pr_{out1}^{PS-AF} as an integration in (40a). The integration cannot be further simplified and is solved numerically in the simulation results.

The outage of the secondary user is given in (D.1b) and is similar to the calculation in proof for Proposition 3. ■

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